International Research Journal of Applied Finance ISSN 2229 – 6891
Vol – III Issue – 3 March, 2012

Index Funds in Small Emerging Financial Markets: The Case of Qatar

Hicham Benjelloun
Associate Professor of Finance
Qatar University
hbenjellou@qu.edu.qa

Khalid Shams Abdulqader
Assistant Professor of Banking
Qatar University

Helmi Hammami
Assistant Professor of Accounting
Qatar University

Abstract

Index funds are widely accepted as the best option for investors seeking diversification. However when the cost of investing is high these benefits can be wiped out. We examine such a possibility in Qatar, a small but emerging financial market where the cost of investing in index funds is high compared to western countries. Using a new approach to diversification we show that index funds in small financial markets are only useful in times of low expectations. In times of high expectations investors are better of buying a small number of stocks directly from the market.

Keywords: Diversification, Optimal Size, Qatar

I. Introduction

The State of Qatar, home of the controversial Al-Jazeera media group and future host of the 2022 FIFA world cup, is a small but wealthy nation that is increasingly influential in the international arena. The country currently relies primarily on its massive reserves of gas and oil but is attempting to diversify its sources of income by encouraging various industries like telecoms, tourism, steel, and others. In addition, it has moved to liberalize its economy, encourage the emergence of the private sector and facilitate the inflow of foreign investments. Qatar is posting massive trade surpluses and despite the financial crisis it has grown consistently at a double digit rate over the past decade. Qatar seems also to be less vulnerable to fluctuations of oil and gas prices than the neighboring gulf monarchies.

The Doha Securities Market (DSM), Qatar's one and only stock market, has experienced explosive growth in capitalization since its inception in 1997. DSM started its operations on May 27, 1997 with 17 listed companies and gradually increased to 43 by the end of 2009. The market started with a market capitalization estimated at six billion Qatari Riyals (QR) and reached QR 335.9 billion by the end of the third quarter of 2009 (1$US=QR3.65). Only Qatari citizens were initially allowed to trade but foreigners have been allowed since April 2005 to trade up to 25 percent of the listed shares in the secondary market. In 2002 Qatar Financial Markets Authority (QFMA) was created as an independent and empowered regulatory and supervisory authority for the capital markets. Table 1 illustrates the evolution of the market since its start through the end of the third quarter of 2009.

Insert Table 1

During 2005 DSM index almost doubled in value compared to 2004 to reach its all time high of 11,053. During 2007, the market index reached 9,581 points, a gain of 2,448 points or 34.32% compared to 2006. During the same year, trading value increased by 45.4% to reach about QR 108.929 billion, trading volume increased by 83% to reach almost 3.5 billion shares, and the number of transactions executed increased by 4.57% to reach 1.8 million transactions. Later the index value decreased substantially.

As of September 2009, four sectors are represented in the Doha Securities Market: Banking (9 companies), Industry (7 companies), Insurance (5 companies), and services (22 companies). A list of companies and their corresponding sectors are shown in Table 2. The Banking and Services sectors are dominating the market with over 80% of the trading value and over 90% of the trading volume.

Insert Table 2

The impressive growth and performance of DSM has led to the emergence of a large mutual fund industry. The leading product offered by the industry is index funds. They are designed to mimic the behavior of the local index which contains the top 20 companies ranked by free-float market

capitalization and daily average traded value and thus provide a fully diversified portfolio to investors.
However the cost of investing in index funds in Qatar is quite expensive. Whereas an index fund in the United States can cost as little as 0.15 percent or less per year the cost can reach 4 to 5 percent in Qatar.
There is no doubt that index funds achieve maximum diversification through risk reduction but high investment costs can potentially counterbalance this benefit. This is precisely what we investigate in this paper. More precisely we attempt to answer the following questions. How best to diversify in Qatar? Is the cost of investing in index funds so high that it would be better to go directly to the stock market? If the answer to the previous question is yes than what would be the optimal number of stocks?
Many factors motivate this paper. First, the Qatari market is young and behave differently from more established markets. Second, there is a lack of research in this area since most of the empirical research about the Middle East has been on the efficiency of the stock markets. Third, the majority of investors in Qatar are inexperienced and unaware of the virtues of diversification and how best to accomplish it. As of today, it appears that no research has been conducted in the Qatari market with respect to the role of mutual funds. We bridge that gap.
This paper is organized as follows. The second section provides an overview of the diversification literature. The third section describes the model used to answer the questions raised in this paper. The fourth section describes the data a methodology used. The fifth section presents the results. Finally the last section concludes

II. Litterature Review

Investors who seek diversification can have the choice of either doing it directly or indirectly through a mutual fund. Surprisingly few papers attempt to compare both strategies. Smith and Schreiner (1970) develop a cost comparison model of direct vs. indirect diversification. The results of their investigation suggest that a small investor would find it more rewarding to diversify through a fund and a large investor would be better off diversifying directly in the market. The paper also concludes that the investment size for which the investor is indifferent as to which alternative he uses to obtain diversification increases with the investment horizon length.
In a subsequent paper Schreiner and Smith (1980) explore the impact of Mayday on diversification costs. On May 1, 1975 the practice of fixed brokerage commission for securities trading was ended and negotiated commissions became the norm. Schreiner and Smith use the same cost model as in Smith and Schreiner (1970) and find that, because small investors do not have bargaining power, this new law makes it harder for them to diversify directly. On the other hand, large investors, who usually have bargaining power, can diversify directly with more ease.
The oldest and most cited diversification paper is Evans and Archer (1968). Evans and Archer find that a stable relationship exists between portfolio size and the level of portfolio dispersion. This relationship decreases rapidly to an asymptote. The asymptote approximates the level of systematic variation in the market. The data used in their paper consists of 470 of the securities listed in the Standard and Poor's Index for the year 1958. Observations on each security are taken at semi-annual intervals for the period January 1958 to July 1967. The statistics employed are the geometric mean of the ex post returns and the standard deviation of the logarithms of the geometric returns. The hypothesis that portfolio standard deviation decreases to an asymptote as diversification increases is tested using the regression function:

$$Y = \frac{B}{X} + A,$$

where

X = the portfolio size,
Y = mean portfolio standard deviations at each level of X,
A and B are constants.

This function yields a very good fit ($R^2 = .9863$). These results are confirmed using t-tests and F-tests. The t-tests test for the reduction of successive mean portfolio standard deviation and the F-tests test for the reduction of standard deviations about the mean portfolio standard deviation. Overall the results of Evans and Archer (1968) raise doubts concerning the justification of increasing portfolio sizes beyond 10 securities since there appears to be no marginal benefit from increasing portfolio size at this level. Evans and Archer's paper is innovative because for the first time the question of portfolio size is addressed.

Solnik (1974) uses the Evans and Archer approach and adds international data in his investigation. The author looks at diversification in major European stock markets and the NYSE. The seven countries are the United Kingdom, Germany, France, Switzerland, Italy, Belgium and The Netherlands. A major finding in Solnik's paper is that the American market, because of its larger size and correlation structure, offers more opportunities for diversification than most of the European markets. When the entire data is blended, there is an even greater reduction in risk compared to the most attractive market (NYSE). This proves that the gains from international diversification are far from negligible. For example, Solnik finds that an internationally well-diversified portfolio is half as risky as the U.S. market. This risk reduction is even more dramatic if a European market is compared to an international portfolio.

Campbell, Lettau, Malkiel and Xu (2001) trace the development of volatility of individual stocks, industries and the overall market from 1962 to 1997. Their data consists of the entire NYSE-NASDAQ-AMEX universe. Campbell et. al come to the following conclusions: The volatility of individual stocks has risen over time, the correlation among stocks returns has fallen over time, the volatility of the market and most of the industries have not changed, and the number of stocks necessary to achieve diversification has increased. The methodology used to confirm the implication that the number of stocks necessary to achieve diversification has increased is identical to Evans and Archer's.

O'Neil (1997) attempts to find out how many funds constitute a diversified mutual fund portfolio. He runs simulations using quarterly mutual funds returns collected. All the mutual funds are categorized as growth or growth and income. The three variables in his analysis are objective (growth or growth and income), holding period (5, 10, 15 or 19 years), and number of funds (1-8, 10, 12, 14, 16, 18, 20, 25, or 30). On average, a growth fund holds 78 securities. It is not surprising therefore that O'Neil finds that the times series standard deviation, which is the method used by Evans and Archer (1968), Solnik (1974), and Campbell, Lettau, Malkiel and Xu (2001), ceases to decrease after the first fund. However when O'Neil uses a cross sectional measure of risk, standard deviation of terminal wealth, he finds a significant decrease of risk after including multiple funds. Although the expected terminal wealth doesn't seem to be impacted by the number of funds, the terminal wealth standard deviation decreases to between 31 percent and 41 percent for growth funds and to between 47 percent and 52 percent for growth and income funds. Longer holding periods requires more funds to achieve diversification because wealth tends to be more dispersed. O'Neil's paper is interesting because it implies that if

a cross sectional measure of risk is used rather than a times series one to investigate the size of a diversified portfolio, the outcome is different.

Statman (1987) criticizes the findings of Evans and Archer and provides a new approach. Statman, unlike previous work, tries to answer the question using asset pricing theory. The approach in his paper is to contrast the marginal benefits and marginal costs. Portfolio size can be increased as long as the marginal benefits exceed the marginal costs.

The 500-stock line (capital market line), where all portfolios $P(n)$ lie, is defined as follow:

$$E[R_{P(n)}] = (R_F + \alpha) + \left\{ \frac{E[R_{P(500)}] - (R_F + \alpha)}{\sigma_{P(500)}} \right\} \sigma_{P(n)},$$

where

$E[R_{P(n)}]$ = expected return of portfolio $P(n)$,

R_F = risk-free rate,

α = excess of the borrowing rate over the lending,

$E[R_{P(500)}]$ = expected return of the 500-stock portfolio,

$\sigma_{P(n)}$ = standard deviation of portfolio $P(n)$, and

$\sigma_{P(500)}$ = standard deviation of the 500-stock portfolio.

To compare the benefits of diversification, a portfolio of n randomly selected stocks, G(n), is compared to a portfolio P(n) that lies on the 500-stock line and has a standard deviation identical to that of portfolio G(n). In general, $E[R_{P(n)}] - E[R_{G(n)}]$ can be interpreted as the benefit from increasing the number of stocks in a portfolio from n to 500. This benefit is then compared to the cost of investing in funds, also known as total expense ratio, that mimic the S&P 500 index. Assuming that no costs are incurred in buying, selling, and holding of portfolios G(n) composed of less than 500 stocks, a leveraged 500-stock portfolio, P(n), is preferable to a portfolio G(n) if the costs of P(n) are lower than the benefits that come with increased diversification. Based on this criterion Statman concludes that the appropriate size is 30 stocks for a borrowing investor and 40 stocks for a lending investor. Using more recent data Statman (2004) finds that more than three hundred stocks are needed to achieve diversification.

More recently Domian, Louton, and Racine (2007) find that one hundred stocks are not enough to reach diversification. They reach that result by examining short fall risk which the possibility of ending wealth being below a target. They conclude also that diversification across industries helps reduce risk but not as much as a random increase of portfolio size.

Benjelloun (2010) reexamines the diversification level of the American market using Evans and Archer approach but with a different method of estimating the asymptote. Benjelloun finds surprising results that seem to go against the current trend in the diversification literature. He concludes that no more than 40 to 50 stocks are needed to achieve diversification. That result if confirmed by future research can have implications on the motives behind the existence of many mutual funds products in the US.

The methodology we use this paper has been inspired by all the papers mentioned above and thus is a synthesis of the diversification literature. It is described in details in the following section.

III. The Model

Investors who choose to invest through an index fund pay an annual cost known as total expense ratio or management fee. This cost is usually a percentage of wealth. Some funds, Vanguard for

example in the USA, charge their customers 0.18% and other funds in the Middle East can charge as much as 5%. For the sake of simplicity the model below assumes that the investor is following a buy and hold strategy.

The variables of the model are as follow:

N = size of a well diversified portfolio,

T = investment horizon length,

W_t = investor's Wealth at time t; t = 0, 1…, T,

r_t^N = expected return of a random portfolio of size N and,

M = Total expense ratio from diversifying indirectly (expressed in percentage).

Let's assume that the investor who diversifies indirectly invests in an index fund. He starts with an initial wealth of W_0. At time 1, wealth has grown at a rate of r_1^{index}. Therefore wealth at time 1 is:

$W_0(1+r_1^{index})$.

After including the funds yearly expenses wealth at time 1 becomes:

$W_1 = W_0(1-M)(1+r_1^{index})$.

Similarly at time 2:

$W_2 = W_0(1-M)^2(1+r_1^{index})(1+r_2^{index})$.

Finally terminal wealth is given by:

$$W_T = W_0(1-M)^T \prod_{t=1}^{T}(1+r_t^{index}). \quad (1)$$

Terminal wealth needs to be adjusted for risk. The following ratio is defined:

$$RVI = \frac{W_T}{\sigma_{index}},$$

where,

RVI = reward-to-variability from indirect diversification, and

σ_{index} = times series standard deviation of the index over time [0,T].

This leads to the following expression for the reward-to-variability from indirect diversification:

$$RVI = \frac{W_0(1-M)^T \prod_{t=1}^{T}(1+r_t^{index})}{\sigma_{index}} \quad (2)$$

The investor who diversifies directly sets up a portfolio of size N. The investor starts with an initial wealth of W_0. At time 1, wealth grows at a rate of r_1^N:

$W_1 = W_0(1+r_1^N)$.

At the end of the second year wealth is given by:

$W_2 = W_1(1+r_2^N)$,

which can be rewritten as:

$W_2 = W_0(1+r_1^N)(1+r_2^N)$.

Finally terminal wealth is given by:

(3) $$W_T = W_0 \prod_{t=1}^{T} (1 + r_t^N).$$

Terminal wealth needs to be adjusted for risk. The following ratio is defined:

$$RVD_N = \frac{W_T}{\sigma_N},$$

RVD_N = reward-to-variability from direct diversification in a portfolio of size N, and

σ_N = times series standard deviation of the portfolio of size N.

This leads to the following expression for the reward-to-variability:

(4) $$RVD_N = \frac{W_0 \prod_{t=1}^{T} (1 + r_t^N)}{\sigma_N}.$$

The following ratio is defined:

$$ratio_N = \frac{RVD_N}{RVI},$$

where,

$ratio_N$ = ratio of reward-to-variability from direct diversification in a portfolio of size N to reward-to-variability from indirect diversification. Direct diversification through a portfolio of size N is better than indirect diversification if and only if $ratio_N \geq 1$. The breakeven point is reached when $ratio_N$ equals one. $ratio_N$ is given by:

$$ratio_N = \frac{\sigma_{index}}{\sigma_N} \frac{W_0 \prod_{t=1}^{T} (1 + r_t^N)}{W_0 (1 - M)^T \prod_{t=1}^{T} (1 + r_t^{index})}.$$

As in Statman (1987) we assume that random portfolios of any size are expected to grow at the market rate. This collapses ratio to:

(5) $$ratio_N = \frac{\sigma_{index}}{\sigma_N} \frac{1}{(1 - M)^T}$$

Formula (5) shows that a comparison between direct and indirect diversification, as measured by $ratio_N$, has to account for portfolio risk (σ_N), mutual fund yearly expenses (M), Investment horizon length (T) and Market risk (σ_{index}). For any portfolio size if $ratio_N$ is less than one that an investor will benefit more from investing in an index fund. If $ratio_N$ is greater than one direct diversification is better.

To investigate the effect of the total expense ratio on $ratio_N$, a first derivative of $ratio_N$ with respect to M needs to be calculated. From (5) it can be shown that:

(6) $$\frac{\partial ratio_N}{\partial M} = \frac{\sigma_{index}}{\sigma_N} \frac{T}{(1 - M)^{T+1}}$$

$\frac{\partial ratio_N}{\partial M}$ is clearly positive. This means that an increase in the expense ratio favors direct diversification. The impact of the expense ratio is considerable and increases with time.

To investigate the effect of the investment horizon length on ratio, a first derivative of $ratio_N$ with respect to T needs to be calculated. From (5) it can be shown that:

(7) $$\frac{\partial ratio_N}{\partial T} = -\frac{\sigma_{index}}{\sigma_N}\frac{Ln(1-M)}{(1-M)^T}.$$

$\frac{\partial ratio_N}{\partial T}$ is clearly positive. This means an increase in the investment horizon length favors direct diversification.

To investigate the effect of portfolio size, N, on ratio, a first derivative of $ratio_N$ with respect to N needs to be calculated. From (5) it can be shown that:

(8) $$\frac{\partial ratio_N}{\partial N} = -\frac{\sigma_{index}}{\sigma_N^2(1-M)^T}\frac{\partial \sigma_N}{\partial N}.$$

According to previous empirical work (Evans and Archer (1968)), for small values of N, $\frac{\partial \sigma_N}{\partial N}$ is large and negative. This means that $\frac{\partial ratio_N}{\partial N}$ is positive. Intuitively speaking, this means that, initially, as portfolio size increases the benefits of direct diversification compared to indirect diversification increase.

According to previous empirical work, for large values of N, $\frac{\partial \sigma_N}{\partial N}$ is negligible. This implies:

(9) $$\frac{\partial ratio_N}{\partial N} \approx 0.$$

Intuitively speaking, after a portfolio reaches a certain size any addition of new stocks doesn't affect the tradeoff between direct and indirect diversification.

Equation (5) will be used in the simulation described in the following section.

IV. Data and Methodology

The data for this study consists of weekly returns of 33 listed companies in DSM, Doha Securities Market, between August 2004 and June 2009, a total of 252 weeks. The activities of these companies cover all sectors and the chosen period coincides with the emergence and growth of investment funds in Qatar.

The returns are calculated as percentage closing price changes of the last trading day of each week. Repeated sampling with replacement is used to simulate hundred equally weighted portfolios for all sizes and the standard deviation is calculated for each as the average standard deviation of the 100 portfolios.

The biggest and first index fund in Qatar is called Al Watani which is managed by Ansbacher & Co and offered through Qatar National Bank the leading bank in Qatar. A quick look at its prospectus reveals the following costs: subscription fee of 2% of initial amount, redemption fee of 0.5% of any shares sold, annual management fee of 1.5%, annual performance fee of 15% of returns in excess of 10%, and custodian fee of 0.5% per year.

Thus, the higher the performance the higher the fees are. If for example the performance is 12% on average for five year the total cost will amount to approximately 3%. If the performance is

let's say 20% on average for five year the total cost will amount to approximately 4.5%. extreme returns are very common in the Qatari market.

To calculate the value of $ratio_N$ (formula (5)) for every level we will assume four possibilities for the expense ratio (M) 2,3,4, and 5%. The proxy for the market portfolio is the DSM general index.

V. Results

Table 3 and Table 4 show the results. Table 3 shows the same pattern from previous research. the standard deviation decreases as portfolio size increases. This decrease is decreasing as portfolio size increases, this is shown in the last column of the table. As portfolio size moves from 1 to 2 stocks risk is reduce by almost 23%, similarly as portfolio size increases beyond 14 risk reduction is consistently below 1%.

Insert Table 3

Using the model described above and the results of Table 3 we highlight the main results of this paper in Table 4. One can clearly see that the size of a well diversified portfolio varies from 6 when the fees are as high as 5% to 13 when the fees are as low as 2%. In an index fund with a fee structure similar to Al Watani's, the fees are high at times when the index performance is high and are close to 2% when the market yields low returns or losses.

Insert Table 4

Thus one can conclude from Table 4 that since it is not difficult for any investor to choose and hold six stocks, direct diversification is the better option at times when the market is performing well because the index fund fees are too high. On the other hand when the market is providing low returns index funds are the better option because it is difficult for a naïve investor to hold as many as 13 stocks or more and maintain them. The index fund contains only twenty stocks after all.

VI. Conclusion

In this paper we investigated the usefulness of index funds in the Qatari financial markets. Index funds are widely accepted as a necessary and useful diversification tool. However when the cost of investing in them is too high, like in Qatar, one start wondering if these costs can overshadow the diversification benefits.

Using a new diversification model that incorporates several variables we find that index funds are useful when the market yields low returns (less than 10%). Bullish investors, on the other hand, will be better of investing directly in the market to avoid high fees.

Since its inception DSM went through many highs and lows. Unfortunately index funds thrived at the wrong time when returns were very high. In times when returns were low the investors wrongfully deserted index funds.

References

Benjelloun, H., (2010), " Evans and Archer, forty years later", Investment Management and Portfolio Innovation, Vol. 7 No. 1, pp. 98-104.

Campbell, J.Y, Lettau, M., Malkiel, B.G. and Xu, Y. (2001), "Have individual stocks become more volatile? An empirical exploration of idiosyncratic risk", Journal of Finance, Vol. 56 No. 1, pp. 1-43.

Dale, L.D., David A.L., and Marie D.R. (2007), « Diversification in portfolios of individual stocks: 100 stocks are not enough", The Financial Review, Vol. 42, pp. 557-570.

Evans, J.L., and Archer, S.H. (1968), "Diversification and the reduction of dispersion: an empirical analysis", Journal of Finance, Vol. 23 No. 5, pp. 761-767.

O'Neal, E.S. (1997), "How many mutual funds constitute a diversified mutual fund portfolio?", Financial Analysts Journal, Vol. 53 No. 2, pp. 37-46.

Schreiner, J.C., and Smith, K.V. (1980), "The impact of Mayday on diversification costs", Journal of Portfolio Management, Vol. 6, pp. 28-36

Smith, K.V., and Schreiner, J.C. (1970), "Direct vs. indirect diversification" Financial Analysts Journal, Vol. 26, pp. 33-38

Solnik, B.H. (1974), "Why not diversify internationally rather than domestically?", Financial Analysts Journal, Vol. 30 No. 4, pp. 48-54.

Statman, M. (1987), "How many stocks make a diversified portfolio?", Journal of Financial and Quantitative Analysis, Vol. 22 No 3. , pp. 353-363.

Statman, M. (2004), "The Diversification Puzzle", Financial Analysts Journal, Vol. 60 No. 4, pp. 44-53.

Table 1: Trading History of DSM

Years	Number of Listed Companies	Market Capitalization (Billion QR)	Index	Trading Volume (Billion QR)
1997	18	6	1,000	0.32
1998	19	Not Available	1,351	0.97
1999	19	20	1,341	1.2
2000	22	18.8	1,233	0.869
2001	23	26.7	1,692	1.5
2002	25	38.4	2,323	3.2
2003	28	97.2	3,946	11.7
2004	30	147.2	6,493.00	23
2005	32	317.2	11,053.00	102
2006	36	221.7	7,133	74.936
2007	40	347.7	9,581	108.929
2008	43	279	6,,886	175,552
End of Q3 2009	43	335.9	7,414	72,382

Table 2: Companies Traded in the DSM

Banking & Financial Sector	
Qat. National Bank	QNBK
Qat. Islamic Bank	QIBK
Comm. Bank Of Qatar	CBQK
Doha Bank	DHBK
Al-Ahli Bank	ABQK
Intl. Islamic Bank	QIIK
Rayan	MARK
First Finance	FFCK
Al Khalij Commercial Bank	KCBK
Insurance Sector	
Qat. Insurance	QATI
Doha Insurance	DOHI
General Insurance	QGRI
Al-Khaleej Insurance	AKHI

Islamic Insurance	QISI
Industrial Sector	
Ind. Manf. Co.	QIMD
National Cement Co.	QNCD
Industries Qatar	IQCD
Qatar Flour Mills Co	QFMD
United Dev.Company	UDCD
Qatar German Co. Med	QGMD
Gulf Holding	GHCD
Services Sector	
Qatar Telecom	QTEL
Electricity and Water	QEWS
Q-Ship	QSHS
Qatar Fuel Company	QFLS
Real Estate Co.	QRES
Salam Intl. Invst.	SIIS
Qatar Navigation	QNNS
Qatar Technical Insp	QTIS
National Leasing	NLCS
Cinema	QCFS
Qatar Meat and Livestock Co.	QMLS
Gulf Warehousing Co.	GWCS
Nakilat	QGTS
Dlala	DBIS
Barwa	BRES
Medicare	MCGS
Mannai Corp.	MCCS
Aamal	AHCS
Qatar Oman	QOIS
Ezdan	ERES
Islamic Securities	IFSS

Table 3: Standard Deviation of Weekly Portfolio Returns and Rate of Risky Reduction of The Qatari Market for the Period Ranging from August 2004 to June 2009

Portfolio Size	Portfolio Standard Deviation	Rate of Risk Reduction
1	6.80%	
2	5.25%	-22.69%
3	4.54%	-13.50%
4	4.21%	-7.27%
5	3.98%	-5.47%
6	3.82%	-4.08%
7	3.72%	-2.69%
8	3.65%	-1.84%
9	3.61%	-1.12%
10	3.53%	-2.25%
11	3.47%	-1.59%
12	3.44%	-1.06%

13	3.39%	-1.31%
14	3.35%	-1.14%
15	3.33%	-0.70%
16	3.30%	-0.75%
17	3.28%	-0.72%
18	3.26%	-0.62%
19	3.24%	-0.52%
20	3.23%	-0.26%
21	3.21%	-0.69%
22	3.20%	-0.45%
23	3.18%	-0.50%
24	3.16%	-0.73%
25	3.16%	-0.08%
26	3.14%	-0.39%
27	3.14%	-0.22%
28	3.14%	-0.01%
29	3.12%	-0.46%
30	3.11%	-0.52%
31	3.10%	-0.21%
32	3.09%	-0.13%
33	3.09%	-0.24%

Table 4: Value of $ratio_N$, Ratio of Reward-to-Variability from Direct Diversification to Reward-to-Variability from Indirect Diversification, for Various Portfolio Sizes and Expenses Ratios.

	Ratio of Reward-to-Variability from Direct Diversification to Reward-to-Variability from Indirect Diversification, $ratio_N$			
Portfolio Size	**M=2%**	**M=3%**	**M=4%**	**M=5%**
1	0.50	0.53	0.55	0.58
2	0.65	0.68	0.72	0.75
3	0.75	0.79	0.83	0.87
4	0.81	0.85	0.89	0.94
5	0.85	0.90	0.94	0.99
6	0.89	0.94	0.98	**1.04**
7	0.92	0.96	**1.01**	1.06
8	0.93	0.98	1.03	1.08
9	0.94	0.99	1.04	1.10
10	0.97	**1.01**	1.07	1.12
11	0.98	1.03	1.08	1.14
12	0.99	1.04	1.10	1.15
13	**1.00**	1.06	1.11	1.17
14	1.02	1.07	1.12	1.18
15	1.02	1.08	1.13	1.19
16	1.03	1.08	1.14	1.20
17	1.04	1.09	1.15	1.21
18	1.04	1.10	1.15	1.21
19	1.05	1.10	1.16	1.22
20	1.05	1.11	1.16	1.22

21	1.06	1.11	1.17	1.23
22	1.07	1.12	1.18	1.24
23	1.07	1.12	1.18	1.24
24	1.08	1.13	1.19	1.25
25	1.08	1.13	1.19	1.25
26	1.08	1.14	1.20	1.26
27	1.09	1.14	1.20	1.26
28	1.09	1.14	1.20	1.26
29	1.09	1.15	1.21	1.27
30	1.10	1.15	1.21	1.27
31	1.10	1.15	1.21	1.28
32	1.10	1.16	1.22	1.28
33	1.10	1.16	1.22	1.28